Math 135, Calculus 1, Fall 2020

Project 1: Modeling an epidemic.
Due: **Friday, October 2** by 11:59pm.

The Project: You will be building and evaluating different functions modeling the *cumultative number of cases of COVID-19* in New York City between March 1 and May 30.

Components. This project has two components: a spreadsheet and a written report. Accordingly, the questions below have two components:

First, each group has a prepared Google Sheets document. Every problem but the last requires you to work in this spreadsheet, either by graphing or making calculations. This worksheet will be included as part of your project. You may add to or modify this spreadsheet, if needed, as you wish to solve the problems and present your solutions.

Second, each group will need to prepare a written report, clearly written in complete sentences. Each problem includes a set of "discussion questions". As you are addressing each problem, keep these questions in mind, and in your report, please reflect and answer these questions.

Setup. When modeling epidemics, an important variable is the *basic reproduction number* R_0. The basic reproduction number is the expected (or average) number of new cases generated by a single infected individual. This (positive) number is effected by many factors, including biological conditions of the pathogen, environmental conditions, and the behavior of the infected population,

Throughout this project, we make the following assumptions:

(i) Each infected individual infects R_0-many new individuals over the course of **one week** (the approximate time duration an infected person is contageous).

(ii) R_0 is constant over each time period in consideration.

Part A: Building a simple model using R_0

Your first set of problems involve building a simple model for the spread of a disease based on the initial population Q_0 and a constant basic reproduction number R_0.

(1) Suppose we have an initial infected population of 1000 people. Compute the total infected population, weekly, for 10 weeks, for the following values of R_0: 0.5, 1, 2, 4. Present this data graphically in Sheet One of your project's Google Spreadsheet.

Discussion questions:
- Is the total infected population function increasing, decreasing, or neither, as a function of time?
- How did you compute these values?

(2) This type of *unrestricted growth* can be modeled using an exponential function $f(x) = A \cdot b^x$. Find a general function $Q(t)$ which models the total infected population as a function of time t (in weeks), with initial population Q_0 and basic reproduction number R_0. Graph two particular examples in Sheet One using the values from Question (1).

Discussion questions:
- Your graph from Part A(1) is individual points, while this graph from Part A(2) is continuous. How might this be a reflection in real-world reporting of pandemics?
- What are the indepedent and dependent variables of your function?

(3) Modify your equation $Q(t)$ so the input is in terms of **days** d as opposed to weeks.

Discussion questions:
- Which do you expect to be more accurate: the daily counts or weekly counts from this model?

Part B: Using your model

You will use your equation $Q(d)$ from Part 1 above to model the spread of COVID-19 in New York in March and April. Throughout, we will be using a funciton $f(d)$ which represents the total number of confirmed COVID-19 cases ffrom the beginning of March to the end of day d.

Sheet Two contains the data and a chart of the total number of reported cases of COVID-19 in New York between March 1 and March 20.

(4) Use your general equation to create a model $Q_1(d)$ for the total COVID-19 cases in New York from March 1 through March 20. That is, choose values of R_0 and Q_0 that you decide best fits this data. Add a graph of your model $Q_1(d)$ to the chart in Sheet Two.[1]

Discussion questions:

- What date does $d = 0$ correspond to? What is the domain of your function?
- Translate the equation $f(10) = 175$ into an explanatory English sentence. Do the same for your value of $Q_1(10)$.
- What are different factors you considered when evaluating different possible choices of R_0 and Q_0?
- How well does the *mathematical model* $Q_1(d)$ represent the *actual* infection count $f(d)$? What are some real-world explainations which may account for this?

New York City went into full lockdown on March 20. You will now investigate how the data changes after this point. Sheet Three contains the data and a chart for March 20 through April 30.

(5) Use your equation $Q_1(d)$ to model the total COVID-19 cases in New York from March 20 through April 30. That is, using the same R_0 value as in Part B(1) and taking $Q_0 = Q_1(20)$, add a graph of your model to the chart in Sheet Three containing the real world data.

Discussion questions:

- What date does $d = 0$ now correspond to?
- How well does $Q_1(d)$ model the spread of COVID-19 in late March and April? What does $Q_1(d)$ predict? (Note that the population of New York City is about 8.3 million.)
- Does the real data qualitatively change after March 20?
- Use the above modeling to conclude whether or not the lockdown was effective in preventing the spread of COVID-19.

(6) Build a new function $Q_2(d)$ to model the spread of COVID-19 between March 20 and April 30; that is, choose new values of R_0 and Q_0 for a new model $Q_2(d)$ to fit the data from March 20 through April 30. Graph your model $Q_2(d)$ on the same set of axes in Sheet Three as in Part B(2).

[1]To add a function to a chart, right click on the chart, select "Data range", click "add series", tap the 4-squares icon, and select your range of values in the spreadsheet.

Discussion questions:

- How does Q_0 for $Q_1(d)$ compare with Q_0 for $Q_2(d)$?
- How does R_0 for $Q_1(d)$ compare to R_0 of $Q_2(d)$? What are some real-world reasons that might be the case?
- How well does $Q_2(d)$ model the data from March 20 through April 30? What can we conclude about the basic reproduction number over this time period?

(7) Use a *line* to model the spread of COVID-19 between March 20 and April 30. That is, decide on values of m and b so that the function $L(d) = md + b$ best fits this data. Graph this model on the same set of axes in Sheet Three as in Part B(2) and B(3).

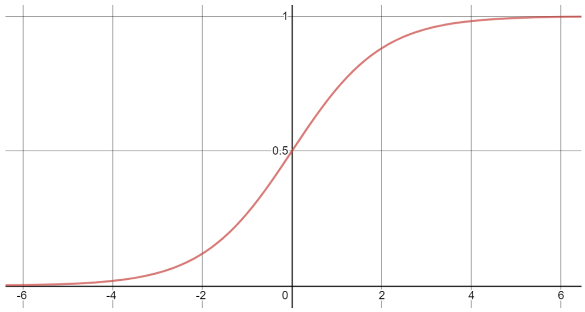
Discussion questions:

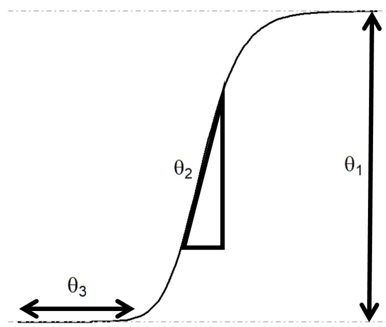
- How did you evaluate your choices of m and b?
- How many cases does your linear model predict for March 20? How does this compare with the real data? Does this bother you? Why or why not?
- Which of $Q_2(d)$ or $L(d)$ better models the spread of COVID-19 over this time period ? Why might this be?
- Linear growth is not unrestricted. What are some scenarios which are better modeled by linear growth?

Part C: Logistic Growth

Epidemics are often accurately modeled by a family of functions with "S"-shaped graphs called **logistic curves**:

$$S(d) = \frac{\theta_1}{1 + e^{-\theta_2 \cdot (t - \theta_3)}}$$





A logistic model has been built into Sheet Four, along with a graph of real-world data from March 1 through May 30.

(8) Build a logistic model to fit this data. That is, choose values for the parameters θ_1, θ_2, and θ_3 that you decide best fit this data.

Discussion questions:

- Why is an "S"-shaped curve more appropriate for an epidemic? What real-world actions may have an influence on the shape of the curve of total infected?
- What are some difference between your logistic model $S(d)$ and the real data? What might account for this discrepency?
- What is the long-term behavior of your model? That is, what is the limit $\lim_{t \to \infty} S(t)$? What does this represent? How does this number compare to the 8.3 million total population in New York City?

Part D: Reflection

(9) Discuss one thing your group has learned during this project.