Math 135, Calculus 1, Fall 2020

09-04: Functions (Section 1.1)

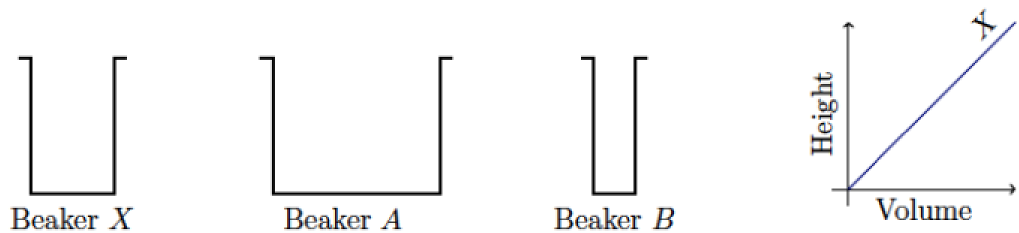
A. Meet your Classmates

Share and discuss with your classmates your "~~Desert Island~~ Quarantine 5":

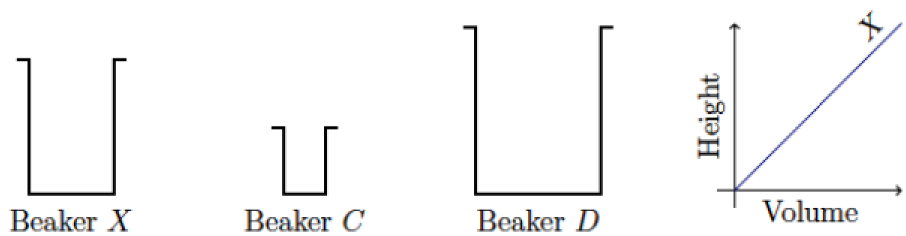
Question 1. *If you were to be isolated on a desert island/apartment/your parent's house (say with an infinite supply of food and water), what* 5 *items would you want to have with you?*

B. Filling Bottles

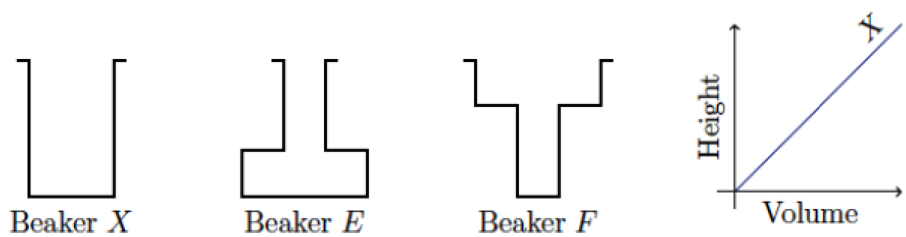
1. The graph below shows how the height of the liquid in beaker X varies as water is steadily dripped into it. On the same diagram, show the height-volume relationship for beakers A and B.



2. Sketch two more for C and D:



3. Sketch two more for E and F:



Exercise 2. Explain your reasoning

(a) Do the diagrams on the previous page describe functions? Justify your answer by providing a definition of function.

(b) What is the independent variable in each of the diagrams above? What is the dependent variable?

(c) How do the domain and range of the relation described by Beaker A compare with the domain and range of the relation described by Beaker X? How is this reflected in the shape of the graph?

How do the domain and range of the relation described by Beaker B compare with the domain and range of the relation described by Beaker X?

(d) How do the domain and range of the relation described by Beaker C compare with the domain and range of the relation described by Beaker X? How is this reflected in the shape of the graph?

How do the domain and range of the relation described by Beaker D compare with the domain and range of the relation described by Beaker X? How is this reflected in the shape of the graph?

(e) How do the domain and range of the relation described by Beaker E compare with the domain and range of the relation described by Beaker X? How is this reflected in the shape of the graph?

How do the domain and range of the relation described by Beaker F compare with the domain and range of the relation described by Beaker X? How is this reflected in the shape of the graph?

C. Properties of Functions

We say that a function f is **increasing** on an interval I if

$$f(x_1) < f(x_2) \quad \text{whenever } x_1 < x_2 \text{ in } I.$$

Likewise, f is **decreasing** on an interval I if

$$f(x_1) > f(x_2) \quad \text{whenever } x_1 < x_2 \text{ in } I.$$

Exercise 3. All of the functions we have seen above are increasing. Given the context, why is this the case? What would it mean for a function to be decreasing?

Exercise 4. Is it possible to create a function $f(x)$ with the following properties? If so, create it. If not, explain why not.

- The domain of $f(x)$ is $[-4, 4]$.
- The range of $f(x)$ is $[-20, 10]$.
- $f(-4) = -6$
- $f(-2) = 8$
- $f(1) = -16$
- $f(x)$ is increasing on the intervals $[-4, -2)$ and $(1, 4]$
- $f(x)$ is decreasing on the interval $(-2, 1)$

We say a function is **even** if $f(-x) = f(x)$ for all x in its domain. The graph of an even function is symmetric about the vertical axis.

Likewise, we say a function is **odd** if $f(-x) = -f(x)$ for all x in its domain. The graph of an odd function is symmetric "about the origin" (the graph is the same after rotating about the origin by 180 degrees).

Exercise 5. Below, sketch the graph of an even function, an odd function, and a function that is neither even nor odd.

Exercise 6. Is it possible to create a function $f(x)$ with the following properties? If so, create it. If not, explain why not.

- $f(x)$ is even.
- The domain of $f(x)$ is $[-4, 4]$.
- The range of $f(x)$ is $[-20, 10]$.
- $f(-4) = -6$
- $f(-2) = 8$
- $f(1) = -16$
- $f(x)$ is increasing on the intervals $[-4, -2)$ and $(1, 4]$
- $f(x)$ is decreasing on the interval $(-2, 1)$

The piecewise definition for $|x|$ is

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Written this way, we see that $|x|$ is an example of a **piecewise function**.

We'll use this definition a lot throughout the course. For example, it's useful for solving inequalities.

Exercise 7. Describe your answers using interval notation and draw them on the number line.

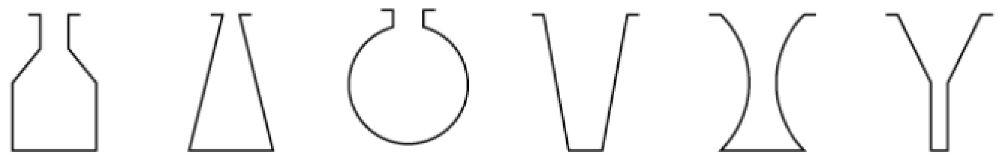
(a) Where is $|2x - 3| \leq 4$?

(b) Where is $|2x - 3| > 4$?

(c) What is the relationship between the answers you found in (1) and (2)?

D. Bonus Bottle Problems

4. Here are six bottles and nine graphs. Choose the correct graph for each bottle. Explain your reasoning clearly. For the remaining three graphs, sketch what the bottles should look like.



Ink bottle, Conical Flask, Evaporating flask, Bucket, Vase, Plugged funnel