Math 135, Calculus 1, Fall 2020

09-21: Limit Law Guide

Suppose that $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ **both exist**. Then:

1. $\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$ (*limit of the sum = sum of the limits*)
2. $\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$ (*limit of the difference = difference of the limits*)
3. $\lim_{x\to a} cf(x) = c\lim_{x\to a} f(x)$ for any constant c (*constants pull out*)
4. $\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$ (*limit of the product = product of the limits*)
5. $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x) \neq 0$ (*limit of the quotient = quotient of the limits*)
6. $\lim_{x\to a} \left[f(x)\right]^n = \left[\lim_{x\to a} f(x)\right]^n$ where n is any positive integer (*this follows from 4.*)
7. $\lim_{x\to a} c = c$ for any constant c (*the limit of a constant is itself*)
8. $\lim_{x\to a} x = a$, and $\lim_{x\to a} x^n = a^n$ where n is any positive integer.
9. $\lim_{x\to a}[f(x)]^{p/r} = [\lim_{x\to a} f(x)]^{p/r}$, where p and r are integers with $r \neq 0$.

One-sided Limits. These rules also follow for one-sided limits.

Limit Existence Theorem. $\lim_{x\to a} f(x) = L$ if and only if $\lim_{x\to a^-} f(x) = L = \lim_{x\to a^+} f(x)$. (*The left- and right-hand limits must both exist and be equal for the general limit to exist.*)

Note: This, along with one-sided limit laws, is helpful when one (or both) two-sided limits DNE.

Direct Substitution Property. If f is a polynomial, rational, exponential, or algebraic function, or is one of $\log_b(x)$, $\cos(x)$, or $\sin(x)$, and a is in the domain of f, then $\lim_{x\to a} f(x) = f(a)$. (*Just plug it in!*)

Simplification Property. If $f(x) = g(x)$ when $x \neq a$, then $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ either both exist or both don't exist, and are equal provided they exist.

The Squeeze Theorem. (Section 2.6) If $f(x) \leq g(x) \leq h(x)$ when x is near a (except possibly at a) and

$$\lim_{x\to a} f(x) = \lim_{x\to a} h(x) = L,$$

then $\lim_{x\to a} g(x) = L$.