# Math 135, Calculus 1, Fall 2020

10-12: Derivative Rules 1

Last week, we introduced the **derivative function** $f'(x)$ of a function $f(x)$, whose evaluation $f'(a)$ at the point $x = a$ is give by:

- the slope of the tangent line at $x = a$
- the instantaneous velocity at time $x = a$

In general, the rule of the derivative function $f'(x)$ can be computed as the limit

$$f'(x) = \lim_{h \to x} \frac{f(x+h) - f(x)}{h}$$

where the numerator is (and remains) a function of $x$. However, this **limit definition of the derivative** can be cumbersome. If only there were some **rules** or **patterns** we could find, that would help us not need to go through the elaborate limit calculations.

We saw the first of these last Friday: the derivative when $f(x) = mx + b$ is a line is just the *constant function* $f'(x) = m$ at the slope of the line. This was proved using the limit definition of the derivative, but now we never need to use it again for a line.

## A. "There's got to be a better way!"

The following formulas can all be computed using the limit definition of the derivative. However, they are **general**, and can then be used when computing the derivative of many different functions.

- **Constant Rule:** $\frac{d}{dx}(c) = 0$
- **Power Rule:** $\frac{d}{dx}(x^n) = nx^{n-1}$ for **any** real number $n$.
- **Constant Multiples:** $\frac{d}{dx}\big(c \cdot f(x)\big) = c \cdot f'(x)$ for any constant $c$
- **Sums and Differences:** $\frac{d}{dx}\big(f(x) \pm g(x)\big) = f'(x) \pm g'(x)$
- **Exponential Functions:** $\frac{d}{dx}(b^x) = \ln b \cdot b^x$, so in particular $\frac{d}{dx}(e^x) = e^x$.

**Exercise 1.** Verify the Power Rule when $n = 3$; that is, use the limit definition of the derivative to show that $\frac{d}{dx}(x^3) = 3x^2$.

**Exercise 2.** Find each of the following derivatives using the power rule:

(a) $\frac{d}{dx}(x^{15})$ (b) $\frac{d}{dx}\left(\frac{1}{x^4}\right)$ (c) $\frac{d}{dx}(\sqrt{x})$ (d) $\frac{d}{dx}(x^{\pi})$

**Exercise 3.** Find $g'(x)$ if $g(x) = 6\sqrt{x} - \frac{3}{x^3} + 5e^x - \pi^4$.

**Exercise 4.** If $p(x) = 4\sqrt[3]{x} + \frac{2}{3}x - \frac{8}{x}$, find the equation of the tangent line to $p$ at the point $x = 8$.

**Exercise 5.** Use the limit definition of the derivative to verify that $\frac{d}{dx}(e^x) = e^x$. You may use the following: $\lim_{x\to 0} \frac{e^x - 1}{x} = 0$.

**Exercise 6.** If the graph of $g(t)$ is a parabola, what type of graph will $g'(t)$ be? Explain.

**Exercise 7.** If $z = e^t + t^e$, find $\frac{dz}{dt}$.

**Exercise 8.** The graph belows shows three functions: $f(x)$, $g(x)$, and $h(x)$. If $f'(x) = g(x)$ and $g'(x) = h(x)$, identify that graph that represents each function. Explain.