Math 135, Calculus 1, Fall 2020

10-22: Implicit Differentiation (Section 3.8)

The **derivative** $f'(x)$ of a function $f(x)$ gives:

- the slope of the tangent line
- the instantaneous velocity
- the instantaneous rate of change

A. Implicit Differentiation

If y and x are related not by a function, but by a general equation

$$F(x, y) = k$$

we can compute the derivative $\frac{dy}{dx}$, the slope of the tangent line at a point, using **implicit differentiation**.

Exercise 1. Compute $\frac{dy}{dx}$ if $x^2 - y^2 + 2xy = 5$.

Exercise 2. Find the slope of the tangent line to $e^{y-x} = 2y^2 - x^2$ at the point $(1, 1)$.

B. Derivative of $\ln(x)$

We can use implicit differentiation to compute the derivatives of **inverse functions**. Recall that a function $f(x)$ is **1-to-1** if each function value $y = f(a)$ is hit exactly one time. In this case, there is an inverse function $f^{-1}(x)$ such that

$$f^{-1}\big(f(x)\big) = x$$

or equivalently that the left equation holds exactly when the right equation holds:

$$f(a) = b \qquad a = f^{-1}(b)$$

Example 1. The function $f(x) = e^x$ and $f^{-1}(x) = \ln(x)$ are inverse functions. More generally, b^x and $\log_b(x)$ are inverse functions.

Exercise 3. Compute the derivative of $y = \ln(x)$ using implicit differentiation:

(i) Consider the equivalent equation $e^y = \ln(x)$, and find $\dfrac{dy}{dx}$.

(ii) Substitute x back in, so that the resulting function for $\dfrac{dy}{dx}$ is only in terms of x (no y's allowed!).