Math 135, Calculus 1, Fall 2020

10-30: Logarithmic Differentiation (Section 3.8)

The **derivative** $f'(x)$ of a function $f(x)$ gives:

- the slope of the tangent line
- the instantaneous velocity
- the instantaneous rate of change

A. Derivatives of inverse trig functions

Exercise 1. Use implicit differentiation to find the derivative of $y = \arctan(x)$:

(a) Use implicit differentiation to find $\frac{dy}{dx}$ for the equivalent equation $tan(y) = x$.

(b) Draw a right triangle with angle y and $\tan(y) = x$.

(c) Use the above triangle to remove all instances of y in your expression for $\frac{dy}{dx}$ from Part (a).

B. Derivative of b^x and $\log_b(x)$

Let $b > 0$. Since e^x and $ln(x)$ are inverse functions, we know in particular that

$$e^{\ln(f(x))} = f(x) \tag{1}$$

for any function $f(x)$.

Exercise 2. Use Equation (1) and the Chain Rule to compute $f'(x)$ for $f(x) = b^x$.

Exercise 3. Use implicit differentiation (as in Exercise 1 to compute $\frac{d}{dx}\Big(\log_b(x)\Big)$.

C. Logarithmic Differentiation

Logarithmic differentiation is a technique which can be used to turn a tedious derivative calculuation, involving lots of Product and Quotient Rules, into a relatively easy procedure.

Exercise 4. Consider the function $f(x) = \dfrac{x(x+1)^3}{(3x-1)^2}$. Gross.

(a) Take the natural log of both sides of this equation, and simplify the right-hand-side by using log rules:

$$\ln(a \cdot b) = \ln(a) + \ln(b), \qquad \ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b), \qquad \ln\left(a^b\right) = b \cdot \ln(a).$$

(b) Use implicit differentiation to compute $f'(x)$ in terms of x and $f(x)$.

(c) Replace $f(x)$ with the original expression to find $f'(x)$ just in terms of x.

Logarithmic differentiation is also useful to compute the derivatives of functions of the form $y = f(x)^{g(x)}$.

Exercise 5. Using the same method as in Exercise 4, compute $f'(x)$ if $f(x) = \sin(x)^{\cos(x)}$.