

Math 135, Calculus 1, Fall 2020

11-02: Logarithmic Differentiation (Section 3.8) and Rates of Change (Section 3.4)

The **derivative** $f'(x)$ of a function $y = f(x)$ gives:

- the slope of the tangent line
- the instantaneous rate of change of y with respect to x

A. LOGARITHMIC DIFFERENTIATION

Exercise 1. Consider the function $y = f(x) = (3x - 2)^{(7x^2+1)}$.

(a) Take the natural log of both sides of this equation, and simplify the right-hand-side by using log rules:

$$\ln(a \cdot b) = \ln(a) + \ln(b), \quad \ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b), \quad \ln\left(a^b\right) = b \cdot \ln(a).$$

(b) Use implicit differentiation on your equation from Part (a) to compute $\frac{dy}{dx}$ in terms of x and y .

(c) Replace y with the original expression for $f(x)$ to find $y' = f'(x)$ just in terms of x .

B. RATES OF CHANGE

For any function $y = f(x)$, the derivative $\frac{dy}{dx}$ measures the **instantaneous rate of change** of y with respect to x .

Example 1. If $T(t)$ measures the temperature T (in degrees Celsius) of an object as a function of time t (in seconds), then $\frac{dT}{dt}$ measures the rate the temperature of the object is changing. The units of $\frac{dT}{dt}$ are $^{\circ}\text{C}/\text{sec}$. If $\frac{dT}{dt} > 0$, then the object is warming; if $\frac{dT}{dt} < 0$, then the object is cooling.

B.1. Application to Economics. Let $C(x)$ be the cost of producing a quantity x of some item, e.g. $C(25) = \$3000$ means it costs \$3000 to produce 25 items. The derivative $C'(x)$ is called the **marginal cost**, and gives an approximation to the cost of producing the $(x + 1)$ -st item. Similarly:

- if $P(x)$ is the profit made from selling x items, then $P'(x)$ is called the **marginal profit**, and
- if $R(x)$ is the revenue made from selling x items, then $R'(x)$ is called the **marginal revenue**.

Exercise 2. Suppose $C(x) = 8000 - 10x + x^2 + 0.01x^3$ represents the cost of producing x computers.

(a) Find the marginal cost function.

(b) Find $C'(10)$ and explain its meaning. What are the units of $C'(10)$?

(c) Find the actual cost of producing the 11th computer. Compare your answer with $C'(10)$.

B.2. Application to Physics. If $s(t)$ is the position of a moving object as a function of time t , then $s'(t) = v(t)$ is the **instantaneous velocity**, and $s''(t) = v'(t) = a(t)$ is the **instantaneous acceleration**. The speed of the object is defined to be $|s'(t)| = |v(t)|$, which is always positive.

Exercise 3. Suppose a particle moves according to the equation $s(t) = t^3 - 12t^2 + 36t$ for $t \geq 0$, where s , the position, is measured in meters, and t , the time, is measured in seconds.

Think of the particle moving along a number line, with s indicating the position on the line.

(a) Compute the velocity and acceleration of the particle at time t .

(b) When is the particle at rest?

(c) What is the particle moving to the right? to the left?

(d) Find the total distance traveled by the particle in the first 6 seconds.