# Math 135, Calculus 1, Fall 2020

11-09: Extreme Values (Section 4.2)

The **derivative** $f'(x)$ of a function $y = f(x)$ gives:

- the slope of the tangent line
- the instantaneous rate of change of $y$ with respect to $x$

## A. Extreme Value Theorem

**Theorem 1** (Extreme Value Theorem on a Closed Interval)**.** *If $f$ is continous on closed interval $I = [a, b]$, that $f$ acheives both an absolute max and an absolute min on $[a, b]$. Moreover, these occur at either critical points or the endpoints $a, b$.*

**Exercise 1.** Find the absolute extreme values of $f(x)$ on the given interval by comparing values at the critical points and endpoints:

(a) $f(x) = x + \sin x$, $I = [0, 2\pi]$

(b) $f(x) = \dfrac{1 - x}{x^2 + 3x}$, $I = [1, 4]$

(c) $f(x) = x \cdot \ln x$, $I = [1, 3]$

**Theorem 2** (Rolle's Theorem). *Suppose $f$ is continuous on $[a, b]$ and differentiable on $(a, b)$. If $f(a) = f(b)$, then there exists a number $c$ between $a$ and $b$ such that $f'(c) = 0$.*

**Exercise 2.** Verify Rolle's Theorem for $f(x) = \sin(x)$ on $[\pi/4, 3\pi/4]$: check that $f(a) = f(b)$, and find the value $c$ in $(\pi/4, 3\pi/4)$ such that $f'(c) = 0$.

**Exercise 3.** Use Rolle's Theorem to prove that $f(x) = x^3 + 3x^2 + 6x$ has precisely one real root:

(a) Find points $x = a$ and $x = b$ such that $f(a) < 0$ and $f(b) > 0$.

(b) By the **Intermediate Value Theorem**, there thus exists some point $c$ in $(a, b)$ with $f(c) = 0$, so $f(x)$ has at least one real root. (We do not need to find the exact value of $x = c$.)

(c) By Rolle's Theorem, what would have to be true about $f$ if it had another root at $x = d$?

(d) Why is the above not possible?