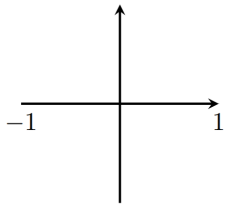
Math 135, Calculus 1, Fall 2020

12-07: Graph Sketching without Technology (Section 4.6)

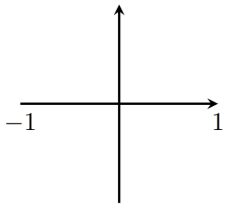
Goal: Combine all of the information obtained from the first and second derivatives (intervals where the function is increasing/decreasing, concave up/down, critical points, extreme values, and inflection points) to sketch a graph of the function.

A. Sketch Snippets

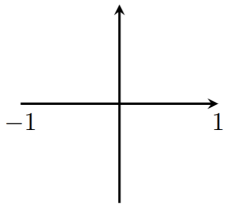
Exercise 1. Draw a sketch of f on the interval $[-1, 1]$ in the following scenarios:



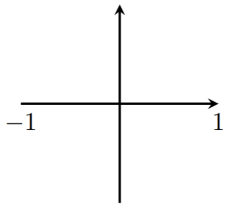
$f(t)$ is positive
$f(t)$ is increasing
$f(t)$ is concave up



$f(t)$ is positive
$f(t)$ is increasing
$f(t)$ is concave down



$f(t)$ is negative
$f(t)$ is decreasing
$f(t)$ is concave up



$f(t)$ is positive
$f(t)$ is decreasing
$f(t)$ is concave down

B. Graph Sketching

Exercise 2. Consider the function $f(x) = 3x^4 - 8x^3 + 6x^2$.

(a) Find the critical points of f.

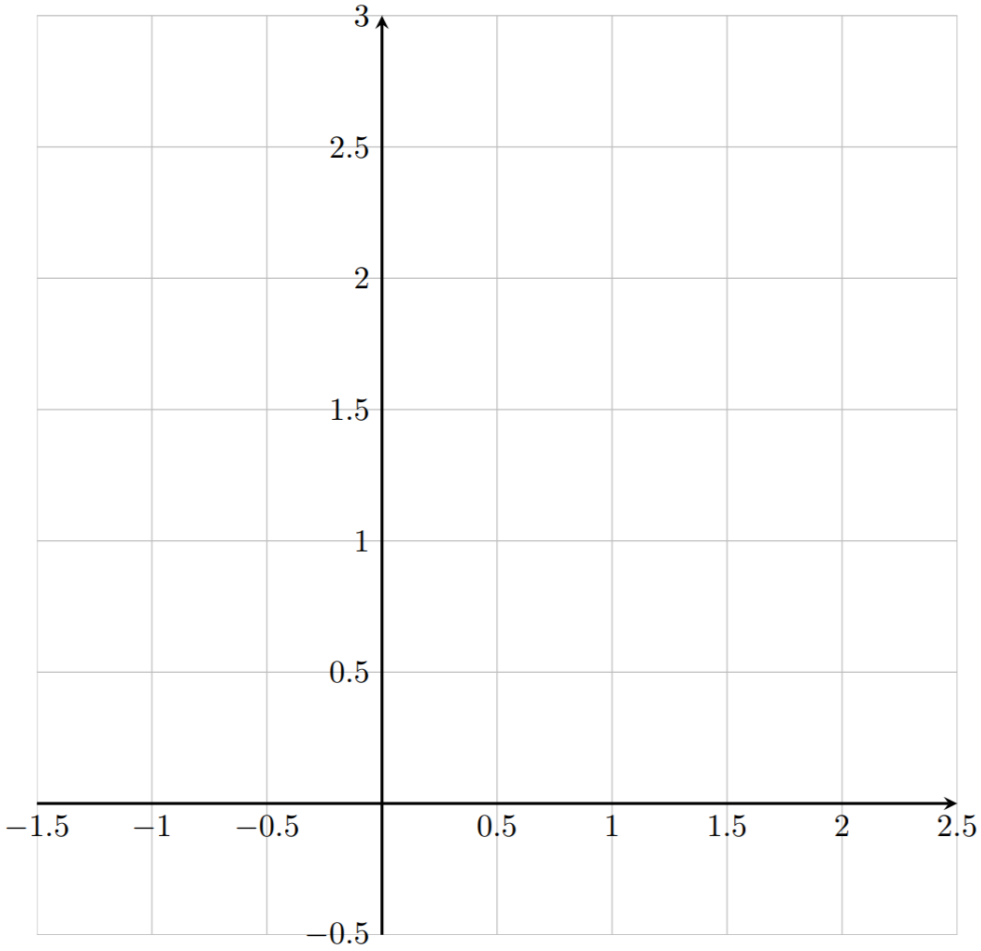
(b) Create a sign chart for the first derivative and determine the open intervals on which the function is increasing/decreasing.

(c) Find the local maxima and minima of f, if any exist. Find the local max/min values by plugging the x-values into the $f(x)$.

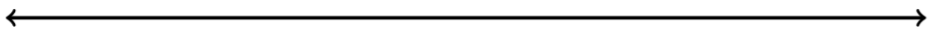
(d) Create a sign chart for the second derivative and determine the open intervals on which the function is concave up/down.

(e) Find any inflection points of f. Find the y-value at each inflection point by plugging the x-values into $f(x)$.

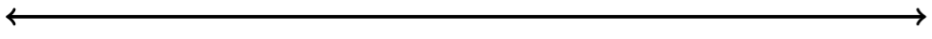
(f) Plot the local extrema and inflection points on the graph. Transfer the information from Parts (b) and (d) to the number lines for $f'(x)$ and $f''(x)$. Finally, sketch the graph of the function $f(x) = 3x^4 - 8x^3 + 6x^2$ using all of this information.



$f'(x)$:



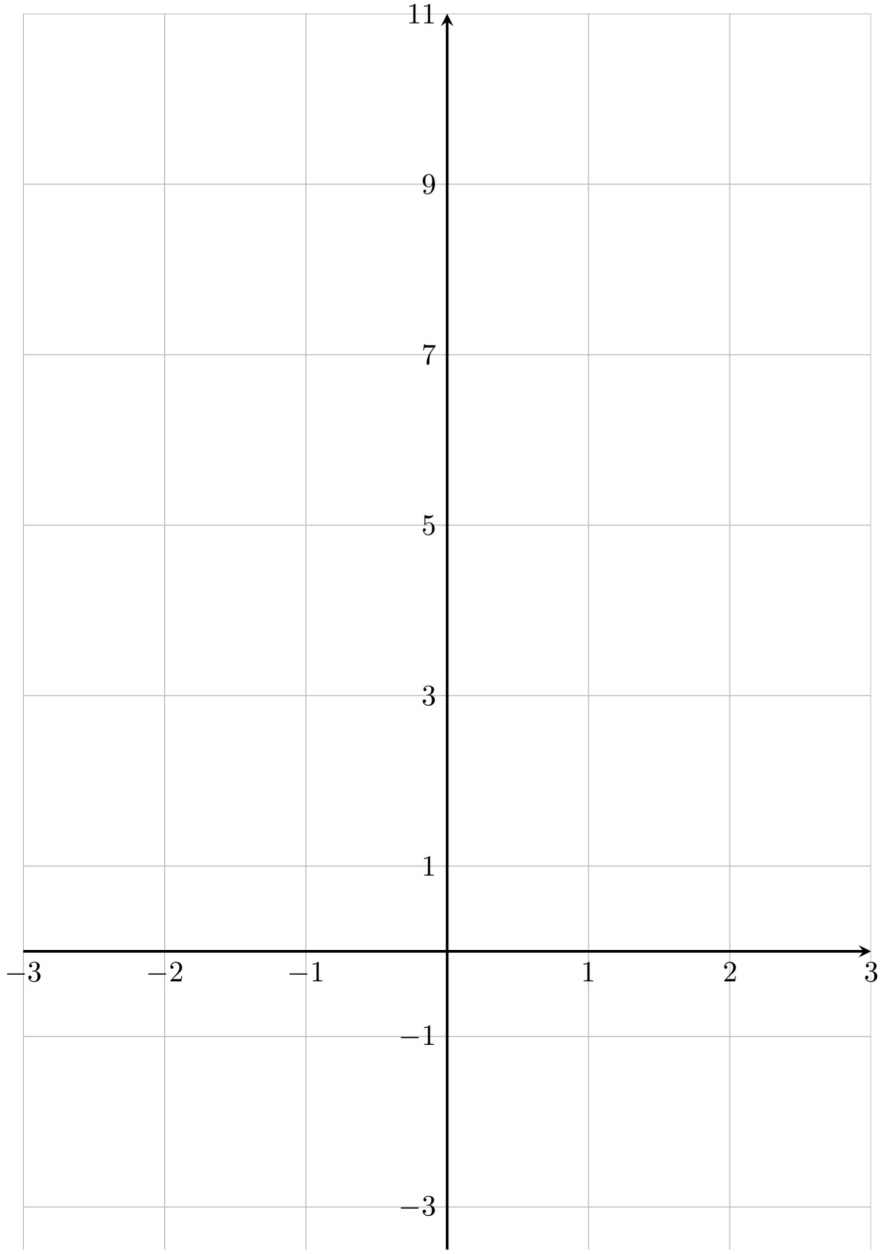
$f''(x)$:



(g) Now use Desmos to get the graph of $y = f(x)$, and compare it to the graph you just drew. How well did you do?

Exercise 3. Using the same process as for Exercise 2, graph $f(x) = x^{1/3}(x + 4)$ on the next page.

Graph of $f(x) = x^{\frac{1}{3}}(x+4)$



$f'(x)$:

$f''(x)$: