Math 135, Calculus 1, Fall 2020

09-04: Functions (Section 1.1)

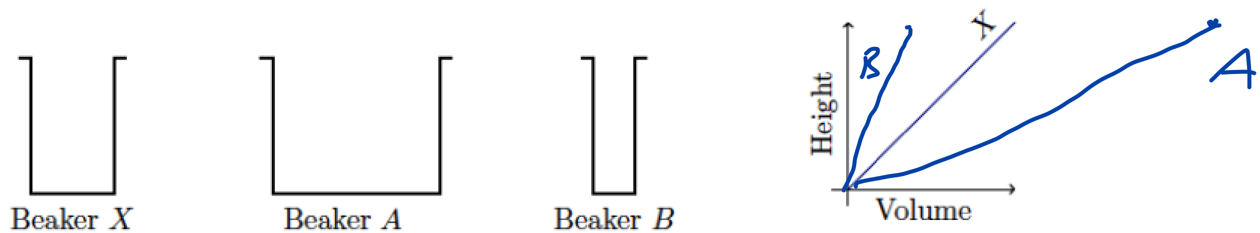
A. Meet your Classmates

Share and discuss with your classmates your "~~Desert Island~~ Quarantine 5":

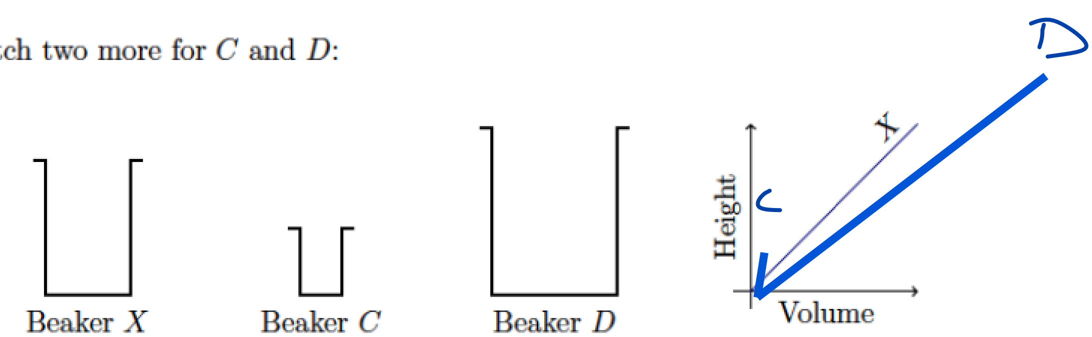
Question 1. *If you were to be isolated on a desert island/apartment/your parent's house* (*say with an infinite supply of food and water*), *what* 5 *items would you want to have with you*?

B. Filling Bottles

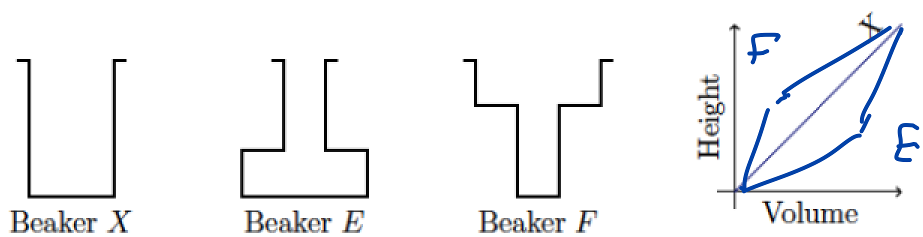
1. The graph below shows how the height of the liquid in beaker X varies as water is steadily dripped into it. On the same diagram, show the height-volume relationship for beakers A and B.



2. Sketch two more for C and D:



3. Sketch two more for E and F:



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Exercise 2. Explain your reasoning

(a) Do the diagrams on the previous page describe functions? Justify your answer by providing a definition of function.

Each input real number gives one output number.

(b) What is the independent variable in each of the diagrams above? What is the dependent variable?

the volume is independent, & the height is dependent

(c) How do the domain and range of the relation described by Beaker A compare with the domain and range of the relation described by Beaker X? How is this reflected in the shape of the graph?

domain is smaller: less possible volume because the beaker is smaller.
range is the same, as the possible heights are the same

How do the domain and range of the relation described by Beaker B compare with the domain and range of the relation described by Beaker X?

domain is bigger: more possible volumes since beaker is larger
range is the same, as the possible heights are the same

(d) How do the domain and range of the relation described by Beaker C compare with the domain and range of the relation described by Beaker X? How is this reflected in the shape of the graph?

domain & range are both smaller, as the height & volume of beaker C are smaller

How do the domain and range of the relation described by Beaker D compare with the domain and range of the relation described by Beaker X? How is this reflected in the shape of the graph?

domain & range are both bigger, as the height & volume of beaker D are bigger

(e) How do the domain and range of the relation described by Beaker E compare with the domain and range of the relation described by Beaker X? How is this reflected in the shape of the graph?

the domain is slightly smaller, as the volume is slightly less
the range is the same, as the heights are the same

How do the domain and range of the relation described by Beaker F compare with the domain and range of the relation described by Beaker X? How is this reflected in the shape of the graph?

the domain is slightly smaller, as the volume is slightly less
the range is the same, as the heights are the same

C. Properties of Functions

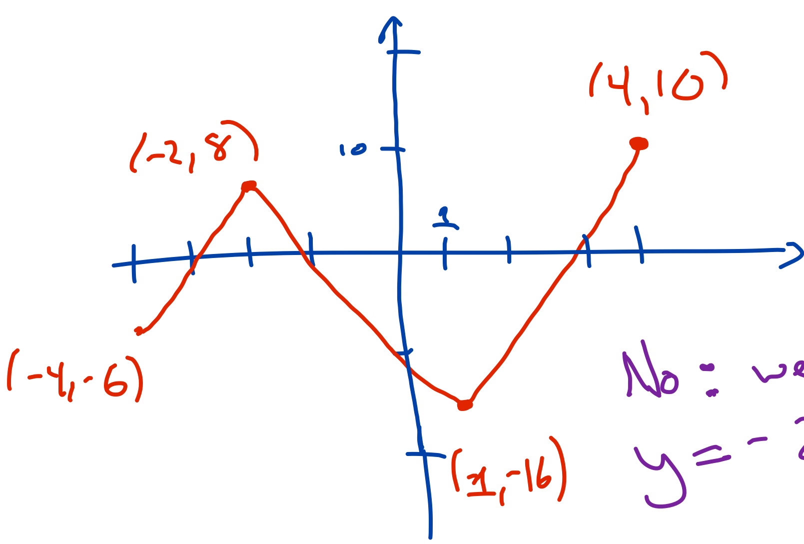
> We say that a function f is **increasing** on an interval I if
> $$f(x_1) < f(x_2) \quad \text{whenever } x_1 < x_2 \text{ in } I.$$
> Likewise, f is **decreasing** on an interval I if
> $$f(x_1) > f(x_2) \quad \text{whenever } x_1 < x_2 \text{ in } I.$$

Exercise 3. All of the functions we have seen above are increasing. Given the context, why is this the case? What would it mean for a function to be decreasing?

the height is always increasing as the volume increases. If a function were decreasing, somehow adding water would have to decrease the height, which is impossible.

Exercise 4. Is it possible to create a function $f(x)$ with the following properties? If so, create it. If not, explain why not.

- The domain of $f(x)$ is $[-4, 4]$.
- The range of $f(x)$ is $[-20, 10]$.
- $f(-4) = -6$
- $f(-2) = 8$
- $f(1) = -16$
- $f(x)$ is increasing on the intervals $[-4, -2)$ and $(1, 4]$
- $f(x)$ is decreasing on the interval $(-2, 1)$

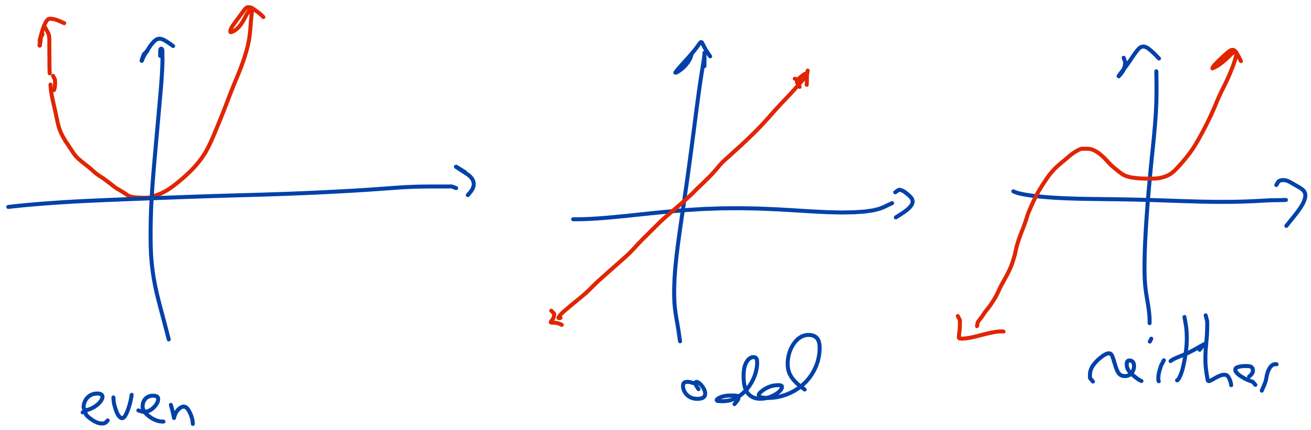


No: we can never hit $y = -20$

We say a function is **even** if $f(-x) = f(x)$ for all x in its domain. The graph of an even function is symmetric about the vertical axis.

Likewise, we say a function is **odd** if $f(-x) = -f(x)$ for all x in its domain. The graph of an odd function is symmetric "about the origin" (the graph is the same after rotating about the origin by 180 degrees).

Exercise 5. Below, sketch the graph of an even function, an odd function, and a function that is neither even nor odd.



Exercise 6. Is it possible to create a function $f(x)$ with the following properties? If so, create it. If not, explain why not.

- $f(x)$ is even.
- The domain of $f(x)$ is $[-4, 4]$.
- The range of $f(x)$ is $[-20, 10]$.
- $f(-4) = -6$
- $f(-2) = 8$
- $f(1) = -16$
- $f(x)$ is increasing on the intervals $[-4, -2)$ and $(1, 4]$
- $f(x)$ is decreasing on the interval $(-2, 1)$

Definitely not:

- if even, inc/dec intervals should flip over y-axis
  (e.g. if f inc on $[-4,-2)$, then f should be dec on $(2,4]$)
  This is clearly not true.
- if odd, inc/dec intervals should be symmetric around the y-axis
  (e.g. if f inc on $[4,-2)$, then f should be inc on $(2,4]$)
  This too is false.

The piecewise definition for $|x|$ is

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Written this way, we see that $|x|$ is an example of a **piecewise function**.

We'll use this definition a lot throughout the course. For example, it's useful for solving inequalities.

Exercise 7. Describe your answers using interval notation and draw them on the number line.

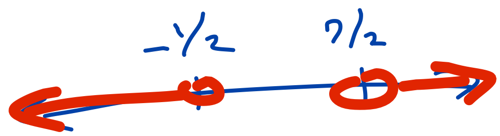
(a) Where is $|2x - 3| \leq 4$?

$2x - 3 \leq 4$ & $2x - 3 \geq -4$

$x \leq \frac{7}{2}$ & $x \geq -1/2$

(b) Where is $|2x - 3| > 4$?

$x \geq 7/2$ or $x < -1/2$

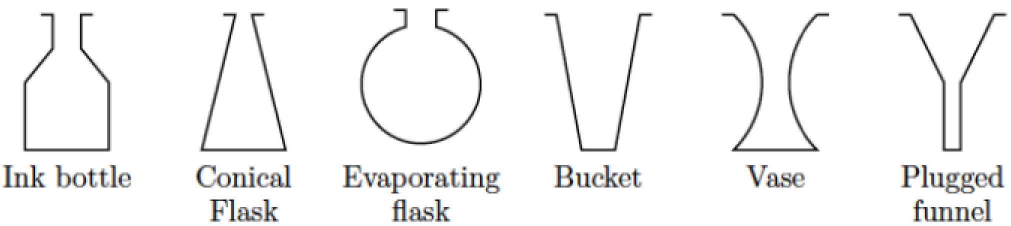


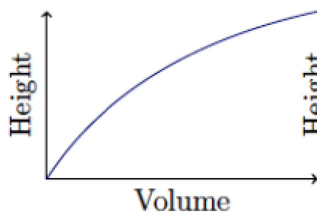
(c) What is the relationship between the answers you found in (1) and (2)?

Together, they cover the whole real line.

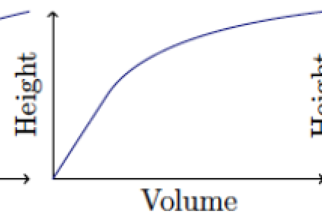
D. Bonus Bottle Problems

4. Here are six bottles and nine graphs. Choose the correct graph for each bottle. Explain your reasoning clearly. For the remaining three graphs, sketch what the bottles should look like.

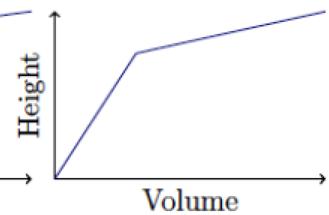




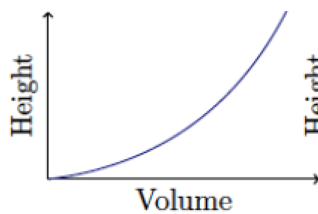
(a)



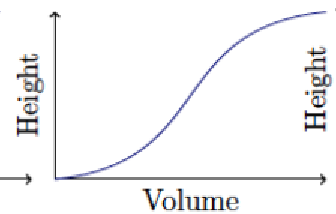
(b)



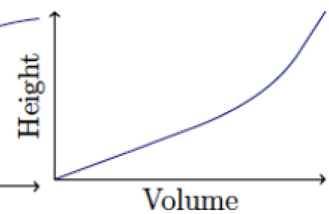
(c)



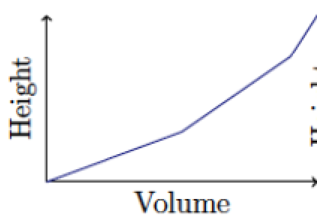
(d)



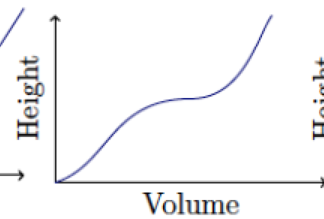
(e)



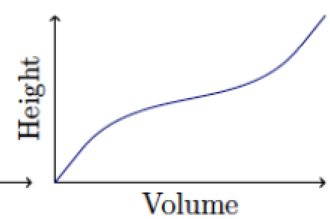
(f)



(g)



(h)



(i)