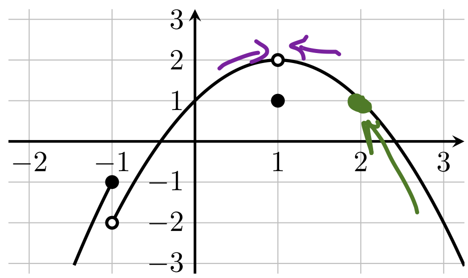
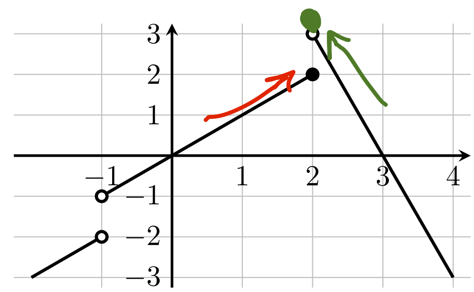
Graphical Limits Using Limit Laws

Graph of f



Graph of g



1. $\lim\limits_{x\to 0}(f(x)+g(x)) = \lim\limits_{x\to 0} f(x) + \lim\limits_{x\to 0} g(x) = 1 + 0 = \boxed{1}$

2. $\lim\limits_{x\to 1}(f(x)g(x)) = \lim\limits_{x\to 1} f(x) \cdot \lim\limits_{x\to 1} g(x) = 2 \cdot 1 = \boxed{2}$

3. $\lim\limits_{x\to 1}(f(x)+g(x)) = \lim\limits_{x\to 1} f(x) + \lim\limits_{x\to 1} g(x) = 2 + 1 = \boxed{3}$

4. $\lim\limits_{x\to 2^+}(2f(x)+3g(x)) = 2 \cdot \lim\limits_{x\to 2^+} f(x) + 3 \cdot \lim\limits_{x\to 2^+} g(x) = 2 \cdot 1 + 3 \cdot 3$

$= 2 + 9 = \boxed{11}$

5. $\lim\limits_{x\to 2^-}(x^2+(\ln x)\cdot g(x)) = \lim\limits_{x\to 2^-}(x^2) + \lim\limits_{x\to 2^-}(\ln(x)) \cdot \lim\limits_{x\to 2^-} g(x)$

$= \boxed{4 + \ln(2) \cdot 2}$

6. $\lim\limits_{x\to 2}(f(x)-g(x))$

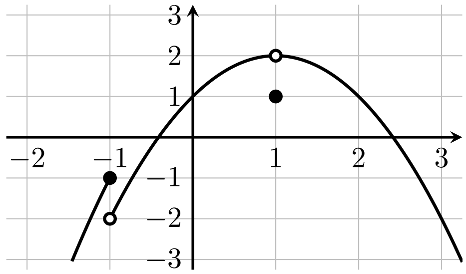
~~$= \lim\limits_{x\to 2} f(x) - \lim\limits_{x\to 2} g(x) = 1 - \text{DNE}$~~

Can't use this limit law since one of the limits DNE

Instead, use one-sided limits:

- $\lim\limits_{x\to 2^-}(f(x)-g(x)) = \lim\limits_{x\to 2^-} f(x) - \lim\limits_{x\to 2^-} g(x) = 1 - 2 = -1$ ← not equal, so limit $\boxed{\text{DNE}}$
- $\lim\limits_{x\to 2^+}(f(x)-g(x)) = \lim\limits_{x\to 2^+} f(x) - \lim\limits_{x\to 2^+} g(x) = 1 - 3 = -2$

Graph of f



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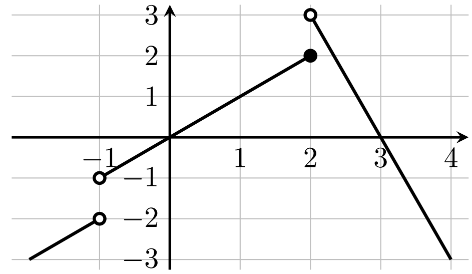


7. $\lim\limits_{x\to 3} \frac{g(x)}{f(x)}$ $= \frac{0}{-2} = \boxed{0}$

8. $\lim\limits_{x\to 3^+} \frac{f(x)}{g(x)}$ when $x \to 3^+$, $g(x)$ is negative towards 0,

so $\lim\limits_{x\to 3^+} \frac{f(x)}{g(x)} = \frac{-2}{-0} = \boxed{+\infty}$

9. $\lim\limits_{x\to 3} \frac{f(x)}{g(x)}$ when $x \to 3^-$, $g(x)$ is positive towards 0,

so $\lim\limits_{x\to 3^-} \frac{f(x)}{g(x)} = \frac{-2}{+0} = -\infty$. $-\infty \neq \infty$ so $\boxed{\text{DNE}}$

10. $\lim\limits_{x\to 1} \sqrt{1 + f(x) + g(x)}$

$= \sqrt{1 + \lim\limits_{x\to 1} f(x) + \lim\limits_{x\to 1} g(x)}$

$= \sqrt{1+2+1} = \sqrt{4} = \boxed{2}$

11. $\lim\limits_{x\to -1} (f(x) + g(x))$

- $\lim\limits_{x\to -1} f(x)$ DNE & $\lim\limits_{x\to -1} g(x)$ DNE X
- Use one-handed limits:

$\lim\limits_{x\to -1^-} (f(x)+g(x)) = \lim\limits_{x\to -1^-} f(x) + \lim\limits_{x\to -1^-} g(x) = -1 + -2 = -3$

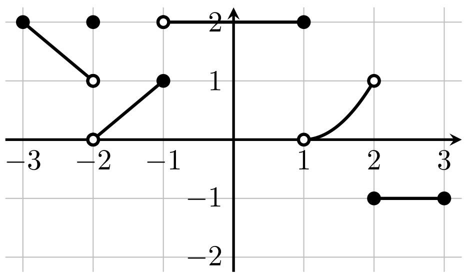
$\lim\limits_{x\to -1^+} (f(x)+g(x)) = \lim\limits_{x\to -1^+} f(x) + \lim\limits_{x\to -1^+} g(x) = -2 + -1 = -3$

Agree so $\boxed{\lim = -3}$

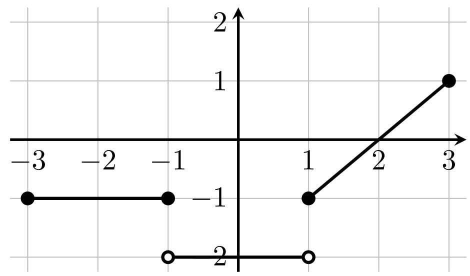
Wacky Limits

Problem: These limits are wacky. Help me understand the key. All I have is the answers and not the reasons why the answers are what they are. Do this by providing the correct mathematical reasons/work explaining how one gets the correct answer.

Graph of f



Graph of g



1. $\lim_{x\to 0}(f(x)+g(x)) = 0$

$= \lim_{x\to 0} f(x) + \lim_{x \to 0} g(x) = 2 - 2 = 0$

2. $\lim_{x\to 2^-}\frac{g(x)}{f(x)} = \lim_{x\to 2^+}\frac{g(x)}{f(x)} = \lim_{x\to 2}\frac{g(x)}{f(x)} = 0$

LHS: $\lim_{x\to 2^-} g(x) \Big/ \lim_{x\to 2^-} f(x) = \frac{0}{+1} = 0$ RHS: $\lim_{x\to 2^+} g(x) \Big/ \lim_{x\to 2^+} f(x) = 0/-1 = 0$

3. $\lim_{x\to -1}(f(x)+g(x)) = 0$

LHS: $\lim_{x\to -1^-} f(x) + \lim_{x\to 1^-} g(x) = 1 + -1 = 0$ RHS: $\lim_{x\to -1^+} f(x) + \lim_{x\to -1^+} g(x) = 2 + -2 = 0$

4. $\lim_{x\to -1}\frac{f(x)}{g(x)} = -1$

LHS $\lim_{x\to -1^-} f(x) \Big/ \lim_{x\to -1^-} g(x) = 1/-1 = -1$ RHS $\lim_{x\to -1^+} f(x) \Big/ \lim_{x\to -1^+} g(x) = 2/-2 = -1$

5. $\lim_{x\to 2}(f(x)g(x)) = 0$

LHS $\lim_{x\to 2^-} f(x) \cdot \lim_{x\to 2^-} g(x) = 1 \cdot 0 = 0$ RHS $\lim_{x\to 2^+} f(x) \cdot \lim_{x\to 2^+} g(x) = -1 \cdot 0 = 0$

6. $\lim_{x\to 3^-} f(g(x)) = 2$

as $x \to 3^-$, $g(x) \to 1^-$, so $\lim_{x\to 3^-} f(g(x)) = \lim_{x\to 1^-} f(x) = 2$

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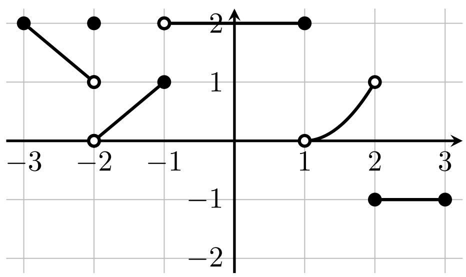

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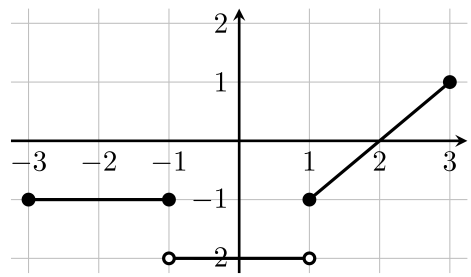

7. $\lim_{x\to 1^+} f(g(x))= 2$

as $x \to 1^+$, $g(x) \to -1^+$, so $\lim_{x\to 1^+} f(g(x)) = \lim_{x\to -1^+} f(x) = 2$

8. $\lim_{x\to -2^-} g(f(x))= -1$ (and NOT -2)

as $x \to -2^-$, $f(x) \to 1^+$, so $\lim_{x\to -2^-} g(f(x)) = \lim_{x\to 1^+} g(x) = -1$

9. $\lim_{x\to 1^-} f(g(x))= 2$ (and NOT 1)

as $x \to 1^-$, $g(x)$ is constant at -2, so $\lim_{x\to 1^-} f(g(x)) = f(-2) = 2$

10. $\lim_{x\to 2^-} \frac{f(x)}{g(x)} = -\infty$

$= {}^{+1}/\boxed{0^-} = -\infty$

as $x \to 2^-$, $g(x) \to 0^-$

11. $\lim_{x\to 2^+} \frac{f(x)}{g(x)} = -\infty$.

$= {}^{-1}/{0^+} = -\infty$

12. $\lim_{x\to 2} \frac{f(x)}{g(x)} = -\infty$

LHS = RHS = $\boxed{-\infty}$