Math 135, Calculus 1, Fall 2020

10-07: The Derivative and Graphs

Last time we introduced the **derivative** $f'(a)$ of a function $f(x)$ at the point $x = a$:

- the slope of the tangent line at $x = a$
- the instantaneous velocity at time $x = a$
- $\lim\limits_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$

Exercise 1. Let $f(x) = 3x^2 - x$. Find $f'(-2)$ by choosing the correct next step from each row below. In the third column, explain why this is the correct step, and what caused the error in the incorrect step.

Step 1.	$\lim\limits_{h\to 0} \dfrac{f(-2+h) - f(-2)}{h}$	$\lim\limits_{h\to -2} \dfrac{f(-2+h) - f(-2)}{h}$	h always goes to 0, not x=a
Step 2.	$\lim\limits_{h\to 0} \dfrac{3(-2+h)^2 + 2 + h + 14}{h}$	$\lim\limits_{h\to 0} \dfrac{3(-2+h)^2 + 2 - h - 14}{h}$	$-f(2)$, & $-(2+h) = -2-h$
Step 3.	$\lim\limits_{h\to 0} \dfrac{3(4 - 4h + h^2) + 2 - h - 14}{h}$	$\lim\limits_{h\to 0} \dfrac{3(4 + h^2) - 2 - h - 14}{h}$	$(a+b)^2$: $a^2+2ab+b^2 \neq a^2+b^2$
Step 4.	$\lim\limits_{h\to 0} \dfrac{12 - 4h + h^2 + 2 - h - 14}{h}$	$\lim\limits_{h\to 0} \dfrac{12 - 12h + 3h^2 + 2 - h - 14}{h}$	$3(4-4h+h^2) = 12-12h+3h^2$
Step 5.	$\lim\limits_{h\to 0} \dfrac{-12h + 3h^2 - h}{h}$	$\lim\limits_{h\to 0} \dfrac{3h^2 - 13h + 12}{h}$	left off "+2−14"
Step 6.	$\lim\limits_{h\to 0} \dfrac{-12\cancel{h} + 3h^2 - h}{\cancel{h}}$	$\lim\limits_{h\to 0} \dfrac{3h^2 - 13h}{h}$	must cancel out an h from all terms in numerator
Step 7.	$\dfrac{3h^2 - 13h}{h}$	$\lim\limits_{h\to 0} \dfrac{h(3h - 13)}{h}$	still need to take the limit
Step 8.	$\lim\limits_{h\to 0}(3h - 13)$	$3h - 13h$	$\parallel$
Step 9.	$-10h$ $3h - 13\cdot h$	-13	$3\cdot 0 - 13 = -13$

where did this h come from?

Exercise 2. Suppose that $g(x) = |x - 3|$. Find $f'(2)$, $f'(3)$, and $f'(4)$. Draw a graph of f and try to make sense of your answers. *Hint: recall the definition of $|x - 3|$ as a piecewise function.*

$|x-3| = \begin{cases} x-3 & x-3 \geq 0, \text{ or } x \geq 3 \\ -(x-3) = -x+3 & x-3 \leq 0, \text{ or } x \leq 3 \end{cases}$

For $f'(4)$, can use this rule for $f(4+h)$, get $\boxed{+1}$

- $f'(2) = \lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{[-(2+h)+3]-[-2+3]}{h} = \lim_{h\to 0} \frac{-h}{h} = \boxed{-1}$
- $f'(3)$: LHS $= \lim_{h\to 0^-} \frac{[-(3+h)+3]-[-3+3]}{h} = \lim_{h\to 0} \frac{-h}{h} = -1$, RHS $= \lim_{h\to 0^+} \frac{[(3+h)+3]-[3-3]}{h} = \lim_{h\to 0^+} \frac{h}{h} = 1$

not equal, so DNE

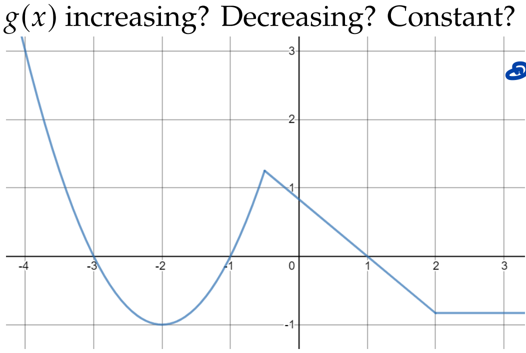
B. Deriative as a function

If we let a vary, the derivative $f'(a)$ is a **new function**.

Definition 1. The **derivative** of $f(x)$ is the function $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$, with domain wherever this limit exists.

- $f'(a) > 0$ exactly when $f(x)$ is **increasing** at $x = a$
- $f'(a) < 0$ exactly when $f(x)$ is **decreasing** at $x = a$

Exercise 3. Suppose the following is the graph of the **derivative** $g'(x)$ of some function $g(x)$. Where is $g(x)$ increasing? Decreasing? Constant?



- Increasing when $g'(x) > 0$: $(-\infty, 3) \cup (-1, 1)$
- Decreasing when $g'(x) < 0$: $(-3, -1) \cup (1, \infty)$

Exercise 4. Suppose $f(x)$ is the function with the following graph. Sketch the graph of $f'(x)$.