Math 135, Calculus 1, Fall 2020

11-04: Rates of Change (Section 3.4)

The **derivative** $f'(x)$ of a function $y = f(x)$ gives:

- the slope of the tangent line
- the instantaneous rate of change of y with respect to x

A. Rates of Change

Exercise 1. Find the rate of change, including units:

(a) Area of a square A in m^2 with respect to the length of a side s in m when $s = 3$.

$A = s^2$

$\frac{dA}{ds} = 2s$ $\qquad \left.\frac{dA}{ds}\right|_{s=3} = 2(3) = \boxed{6 \ m^2/m}$

(b) The diameter d of a circle in cm with respect to the radius r in cm.

$d = 2r$

$\boxed{\frac{d(d)}{dr} = 2 \ cm/cm}$

(c) Volume V in ft^3 with respect to the radius in feet, if the height is equal to the radius.

of a cylinder

$V = \pi r^2 h$, $r = h \Rightarrow V = \pi r^3$

$\boxed{\frac{dV}{dr} = 3\pi r^2 \ ft^3/ft}$

Exercise 2. The dollar cost of producing x bagels, in thousands, is given by the function

$$C(x) = 50x^3 - 750x^2 + 3740x + 3750.$$

(a) What is the cost of producing 4000 bagels?

$C(4) = 9910$

(b) Find the approximate cost of producing ~~the 4001-st bagel~~. the 40001st thru 5000th bagel

$C'(x) = 150x^2 - 1500x + 3740$

$\boxed{C'(4) = 140 \text{ dollars}/1000 \text{ bagels}}$

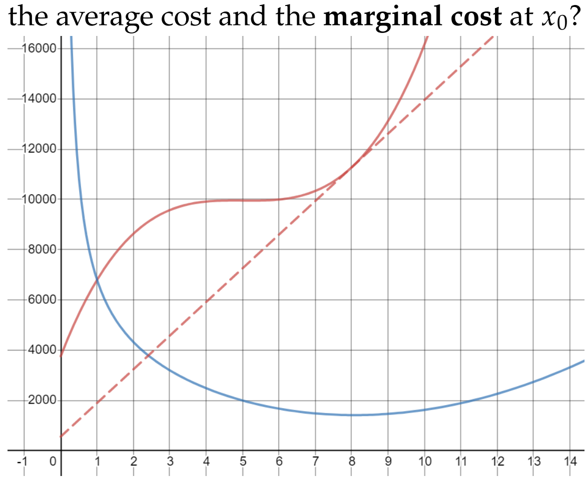
(c) Compare your answer to Part (b) with the actual cost of producing the 4001 st bagel.

$C(5) - C(4) = 9950 - 9910 = \boxed{40 \ \$/1000 \text{ bagels}}$

(d) What is the **average cost** of ~~a~~ 1000 bagels when producing 4000 bagels?

$C(4)/4 = \dfrac{9910}{4} = \boxed{2477.5 \ \$/1000 \text{ bagels}}$

(e) The blue graph below depicts the average cost as a function of x, while the red depicts $C(x)$. At what level of production x_0 is the average cost smallest? What is the relationship between the average cost and the **marginal cost** at x_0?



- avg cost is minimize at $x_0 = 8.075$

- $\dfrac{C(8.075)}{8.075} = 1408$

- $C'(8.075) = 1408$

equal!