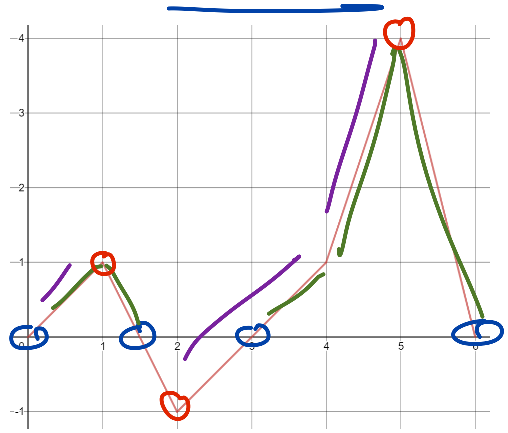
Math 135, Calculus 1, Fall 2020

11-18: Derivative Tests

Exercise 1. Suppose the function f on $[0, 6]$ has **derivative** given by the following piecewise-linear function:



(a) What are the critical points of f?

When $f'(x) = 0$ or DNE: $\boxed{x = 0, 1.5, 3, 6}$

(b) On what interval(s) is f increasing? decreasing? Make a sign chart for the first deriative.

- $f(x)$ inc $\iff f'(x) > 0$: $(0, 1.5) \cup (3, 6)$
- $f(x)$ dec $\iff f'(x) < 0$: $(1.5, 3)$

(c) On what interval(s) is f concave up? down? Make a sign chart for the second derivative.

- $f(x)$ concave up $\iff f''(x) > 0 \iff \frac{d}{dx}(f'(x)) > 0 \iff f'(x)$ inc: $(0, 1) \cup (2, 5)$
- $f(x)$ concave down $\iff f'(x)$ dec: $(1, 2) \cup (5, 6)$

(d) What are the inflection points of f?

- Where concavity changes $\iff$ local extrema of $f'(x)$: $x = 1, 2, 5$

(e) Classify the critical points using **both** the First and Second Derivative Test.

(ignore endpoints $x = 0, 6$)

- Sign line for $f'(x)$:

$$\begin{array}{ccccccc} 0 & & 1.5 & & 3 & & 6 \\ & + & 0 & - & 0 & + & \end{array}$$

So $x = 1.5$ is a max & $x = 1.5$ is a min

- $f''(1.5) =$ slope of $f'(x)$ @ $x = 1.5$ $= -2 < 0 \implies$ local max $\cap$

$f''(3)$: slope of $f'(x)$ @ $x = 3$ $= +1 > 0 \implies$ local min $\cup$

Exercise 2. Suppose $g(x)$ is a function which is continuous at all $x \neq 2$ (where it has a vertical asymptote), with **first** derivative given by

$$g'(x) = \frac{(x+4)(x-1)^2}{x-2}.$$

(a) Find all critical points for g.

$$\frac{(x+4)(x-1)^2}{x-2} = 0$$

$$(x+4)(x-1)^2 = 0$$

$x+4=0$ $\quad$ $\boxed{x=-4}$

$x-1=0$ $\quad$ $\boxed{x=1}$

$x-2=0$ $\quad$ $\boxed{x=2}$

(b) Create a sign chart for the first derivative of g, and classify these critical points.

		-4		1		2	
$f'(x)$	$+$	0	$-$	0	$-$	DNE	$+$

(c) Does g have an absolute max value? absolute min value? Explain.

- No abs min: ~~the~~ sign chart plus $x=2$ vertical asymptote implies that $\lim_{x \to 2} g(x) = -\infty$
- No abs max: $\lim_{x \to \infty} g'(x) = \infty$, so $\lim_{x \to \infty} g(x) = \infty$

Exercise 3. Suppose $h(x)$ is a function which is continuous at all $x \neq 2$ (where it has a vertical asymptote), with **second** derivative given by

$$h''(x) = \frac{(x+2)x^3}{(x-3)^2}.$$

(a) Create a sign chart for the second derivative of h. Find all inflection points for h.

$(x+2)x^3 = 0$

$x+2=0$ $\quad$ $x^3=0$

$\boxed{x=-2}$ $\quad$ $\boxed{x=0}$

$(x-3)^2 = 0$

$x-3=0$

$\boxed{x=3}$

		-2		0		3	
$h''(x)$	$+$	0	$-$	0	$+$	DNE	$+$
	$\cup$		$\cap$		$\cup$		$\cup$

$\boxed{\text{inflection points: } x=-2,\ x=0}$

(b) Suppose $x = 1$ is a critical point for h. Classify this critical point.

$h''(1) > 0 \Rightarrow \boxed{\text{local min}}$