# Math 135, Calculus 1, Fall 2020

12-04: Applied Optimization II

Today we will be applying our optimization techniques (EVT and others) to solve word problems.

- The goal for all of these questions is to translate the word problem into a math question of the form "where is this function of one variable maximized/minimized?" This process has many steps: finding the function at hand (and if necessary converting it into a function of one variable), considering the domain, and applying the EVT (or related analysis).
- Word problems are hard! They are hard for everyone — students, grad students, professors, economists, politicans, doctors — everyone. It is okay to get discouraged or frustrated. But these are the most important questions: applying calculus techniques and problem solving skills to the "real world". No one is going to offer you a job because you can take the derivative of a function, but a good problem solver is indispensable.

**Exercise 1.** League of Legends is a multiplayer online video game. One aspect of the game involves battling other players. A player's *Effective Health* when defending against physical damage is given by

$$E = h + \frac{ha}{100}$$

where $h$ is the player's **Health** and $a$ is the player's **Armor**. Players can purchase more Health and Armor with gold coins: Health costs 2.5 coins per unit, and Armor costs 18 coins per unit. Suppose a player has 2662 gold coins. What is the maximum Effective Health the player can achieve?

(a) Assume the player will spend all of their coins. Write an equation relating $h$ and $a$.

$2.5(3) + 18(5)$

$$2662 = 2.5h + 18a$$

(b) Use your answer to Part (a) to write an equation for the player's Effective Health in terms of their Health $h$.

$$a = \frac{2662 - 2.5h}{18}$$

$$\Rightarrow E(h) = h + \frac{h}{100}\left(\frac{2662-2.5h}{18}\right) = h + \frac{2662}{1800}h - \frac{2.5}{1800}h^2$$

$$\boxed{E(h) = \frac{2231}{900} \cdot h - \frac{1}{720}h^2}$$

(c) What is the domain of this function in the context of this problem?

- both $h, a \geq 0$
- if $a = 0$, then $2662 = 2.5h + 18(0)$

$$h = \frac{2662}{2.5} = \frac{5324}{5}$$

$$\boxed{\left[0, \frac{5324}{5}\right]}$$

(d) Maximize this function.

$$E'(h) = \frac{2231}{900} - \frac{1}{360}h \overset{!}{=} 0$$

$$\Rightarrow \frac{1}{360}h = \frac{2231}{900} \Rightarrow \boxed{h = \frac{2231 \cdot 360}{900} = \frac{4462}{5}}$$

$$E\left(\frac{4462}{5}\right) = \frac{4977361}{4500} \qquad \boxed{1106.08}$$

Absolute max value @ $h = \frac{4462}{5} = 892.4$

$$E(0) = 0$$

$$E\left(\frac{5324}{5}\right) = \frac{5324}{5} \qquad 1064.8$$

(e) How do you know your value from Part (c) is the absolute maximum (as opposed to a local maximum or an absolute/local minimum)?

EVT!

Should buy 892.4 units of health, and

$$\frac{2662 - 2.5(892.4)}{18} = 23.9\overline{4} \text{ units of armor}$$

**Exercise 2.** A food company wants to design aluminum cans which minimize the amount of metal needed. The cans need to hold 12 oz of liquid, or approximately 21.7 $\text{in}^3$.

(1) Write an equation for the surface area $A$ of a cylindrical can in terms of the radius $r$ and the height $h$.

$$A = \underbrace{2\pi r h}_{\text{sides}} + \underbrace{2(\pi r^2)}_{\text{top \& bottom}}$$

(2) Use the constraint that the cans need to hold 12 oz of liquid to write an equation relating $r$ and $h$.

$$21.7 = (\pi r^2) \cdot h$$

(3) Combine Parts (a) and (b) to write an equation for the surface area $A$ as a function of the radius $r$.

$$h = \frac{21.7}{\pi r^2} \Rightarrow A = 2\pi r\left(\frac{21.7}{\pi r^2}\right) + 2\pi r^2$$

$$\boxed{A = \frac{43.4}{r} + 2\pi r^2}$$

(4) What is the domain of this function in this context?

- $r, h > 0$ (if either equal zero, the volume would be $0 \neq 21.7$)
- But can be as big as we want, so

$$\boxed{(0, \infty)}$$

(5) Minimize this function. Justify your answer.

$$A'(r) = \frac{-43.4}{r^2} + 4\pi r^2 \overset{!}{=} 0$$

$$4\pi r^2 = \frac{43.4}{r^2}$$

$$4\pi r^4 = 43.4$$

$$r^4 = 43.4/4\pi$$

$$\boxed{r = \sqrt[4]{43.4/4\pi} \approx 1.363}$$

Other CP: $r = 0$

$$A''(r) = \frac{+86.8}{r^3} + 8\pi r > 0 \text{ when } r > 0$$

$\Rightarrow$ $\boxed{r = 1.363\ldots \text{ is a local and absolute min!}}$

(6) What does your answer to Part (c) mean in the context of this problem?

That this radius length will minimize the amount of aluminum used in a 12oz can

(7) What are the dimensions of the can the company should make?

$$\boxed{\begin{aligned} r &= 1.363\ldots \\ h &= \frac{21.7}{\pi r^2} \approx 3.717 \end{aligned}}$$