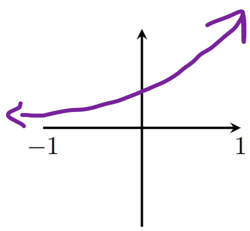
Math 135, Calculus 1, Fall 2020

12-07: Graph Sketching without Technology (Section 4.6)

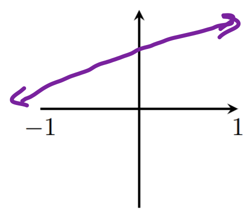
Goal: Combine all of the information obtained from the first and second derivatives (intervals where the function is increasing/decreasing, concave up/down, critical points, extreme values, and inflection points) to sketch a graph of the function.

A. Sketch Snippets

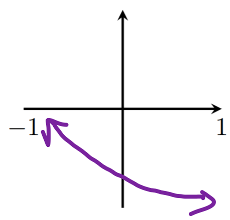
Exercise 1. Draw a sketch of f on the interval $[-1, 1]$ in the following scenarios:



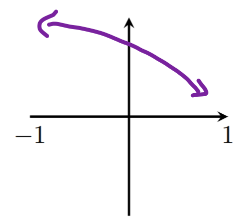
$f(t)$ is positive
$f(t)$ is increasing
$f(t)$ is concave up



$f(t)$ is positive
$f(t)$ is increasing
$f(t)$ is concave down



$f(t)$ is negative
$f(t)$ is decreasing
$f(t)$ is concave up



$f(t)$ is positive
$f(t)$ is decreasing
$f(t)$ is concave down

B. Graph Sketching

Exercise 2. Consider the function $f(x) = 3x^4 - 8x^3 + 6x^2$.

(a) Find the critical points of f.

$$f'(x) = 12x^3 - 24x^2 + 12x \stackrel{!}{=} 0$$
$$12x(x^2 - 2x + 1) = 0$$
$$12x(x-1)(x-1) = 0$$
$$\boxed{x=0} \qquad \boxed{x=1}$$

(b) Create a sign chart for the first derivative and determine the open intervals on which the function is increasing/decreasing.

$f'(-1) = (-)(-)(-) < 0$

$f'(0.5) = (+)(-)(-) > 0$

$f'(2) = (+)(+)(+) > 0$

	-1	0	0.5	1	2
$f'(x)$	$-$	0	$+$	0	$+$

Inc: $(0,1) \cup (1, \infty)$ Dec: $(-\infty, 0)$

(c) Find the local maxima and minima of f, if any exist. Find the local max/min values by plugging the x-values into the $f(x)$.

local min: $x = 0$, $f(0) = 0 - 0 + 0 = 0$ $(0,0)$

other point: $x = 1$, $f(1) = 3 - 8 + 6 = 1$ $(1,1)$

(d) Create a sign chart for the second derivative and determine the open intervals on which the function is concave up/down.

$$f''(x) = 36x^2 - 48x + 12 \stackrel{!}{=} 0$$
$$12(3x^2 - 4x + 1) = 0$$
$$(3x-1)(x-1) = 0$$
$$\boxed{x = 1/3} \qquad \boxed{x=1}$$

		$1/3$		1	
$f''(x)$	$+$		$-$		$+$

$f''(0) = 12 > 0$

$f''(0.5) = (+)(-) < 0$

$f''(2) = (+)(+) > 0$

Concave Up: $(-\infty, 1/3) \cup (1, \infty)$

Concave Down: $(1/3, 1)$

(e) Find any inflection points of f. Find the y-value at each inflection point by plugging the x-values into $f(x)$.

$\boxed{x = 1/3}$

$$f(1/3) = 3\left(\frac{1}{81}\right) - 8\left(\frac{1}{27}\right) + 6\left(\frac{1}{9}\right)$$
$$= \frac{1}{27} - \frac{8}{27} + \frac{18}{27} = \boxed{11/27}$$

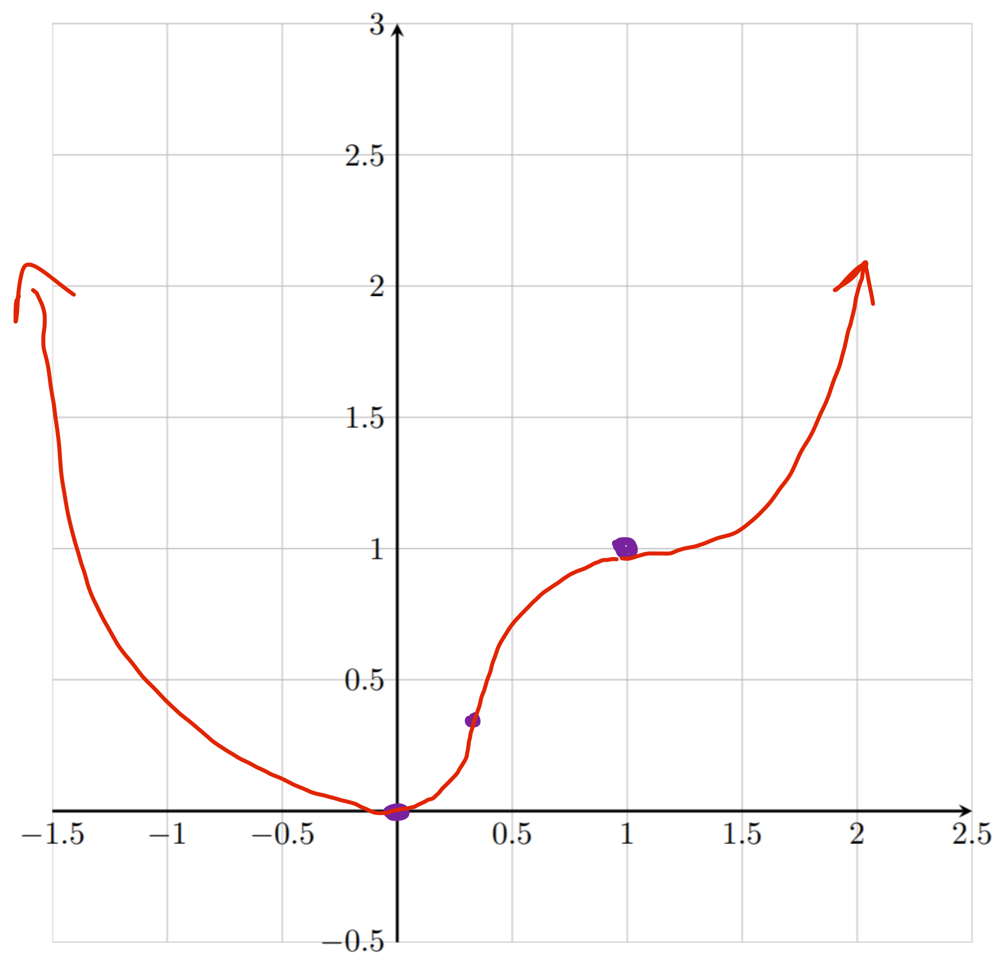
$(1/3, 11/27)$

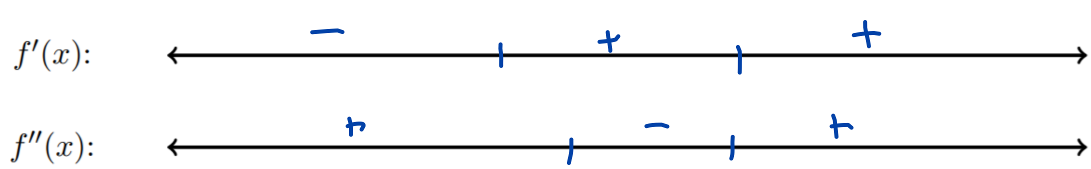
$\boxed{x = 1}$

$f(1) = 1$

$(1,1)$

(f) Plot the local extrema and inflection points on the graph. Transfer the information from Parts (b) and (d) to the number lines for $f'(x)$ and $f''(x)$. Finally, sketch the graph of the function $f(x) = 3x^4 - 8x^3 + 6x^2$ using all of this information.





(g) Now use Desmos to get the graph of $y = f(x)$, and compare it to the graph you just drew. How well did you do?

Exercise 3. Using the same process as for Exercise 2, graph $f(x) = x^{1/3}(x+4)$ on the next page.

$$f'(x) = \frac{1}{3x^{2/3}}(x+4) + x^{1/3} \stackrel{!}{=} 0$$

DNE: $\boxed{x=0}$

$$(x+4) + (3x^{2/3})x^{1/3} = 0$$

$$x+4+3x=0$$

$$4x=-4$$

$$\boxed{x=-1}$$

$\boxed{\begin{array}{l} \cdot f(0)=0 \\ \cdot f(-1)=-3 \end{array}}$

$$f''(x) = \frac{(1)(3x^{2/3}) - (x+4)\left(2/x^{1/3}\right)}{(3x^{2/3})^2} + \frac{1}{3x^{2/3}} = \frac{3x^{2/3} - \frac{2x+8}{x^{1/3}} + 3x^{2/3}}{9x^{4/3}}$$

DNE: $\boxed{x=0}$

$$f''(x) \stackrel{!}{=} 0$$

$$6x^{2/3} - \frac{2x+8}{x^{1/3}} \stackrel{!}{=} 0$$

$$6x^{2/3}(x^{1/3}) - (2x+8) = 0$$

$$6x - 2x - 8 = 0$$

$$4x = 8$$

$$\boxed{x=2}$$

$\boxed{f(2) = 6\sqrt[3]{2}}$

		-1		0		2	
$f'(x)$	$-$	0	$+$	DNE		$+$	
$f''(x)$		$+$		DNE	$-$	0	$+$

$$f'(-2) = \frac{2}{3\sqrt[3]{4}} - \sqrt[3]{2} < 0$$

$$f'(-0.5) = \frac{3.5}{3\sqrt[3]{0.5}} - \sqrt[3]{0.5} > 0$$

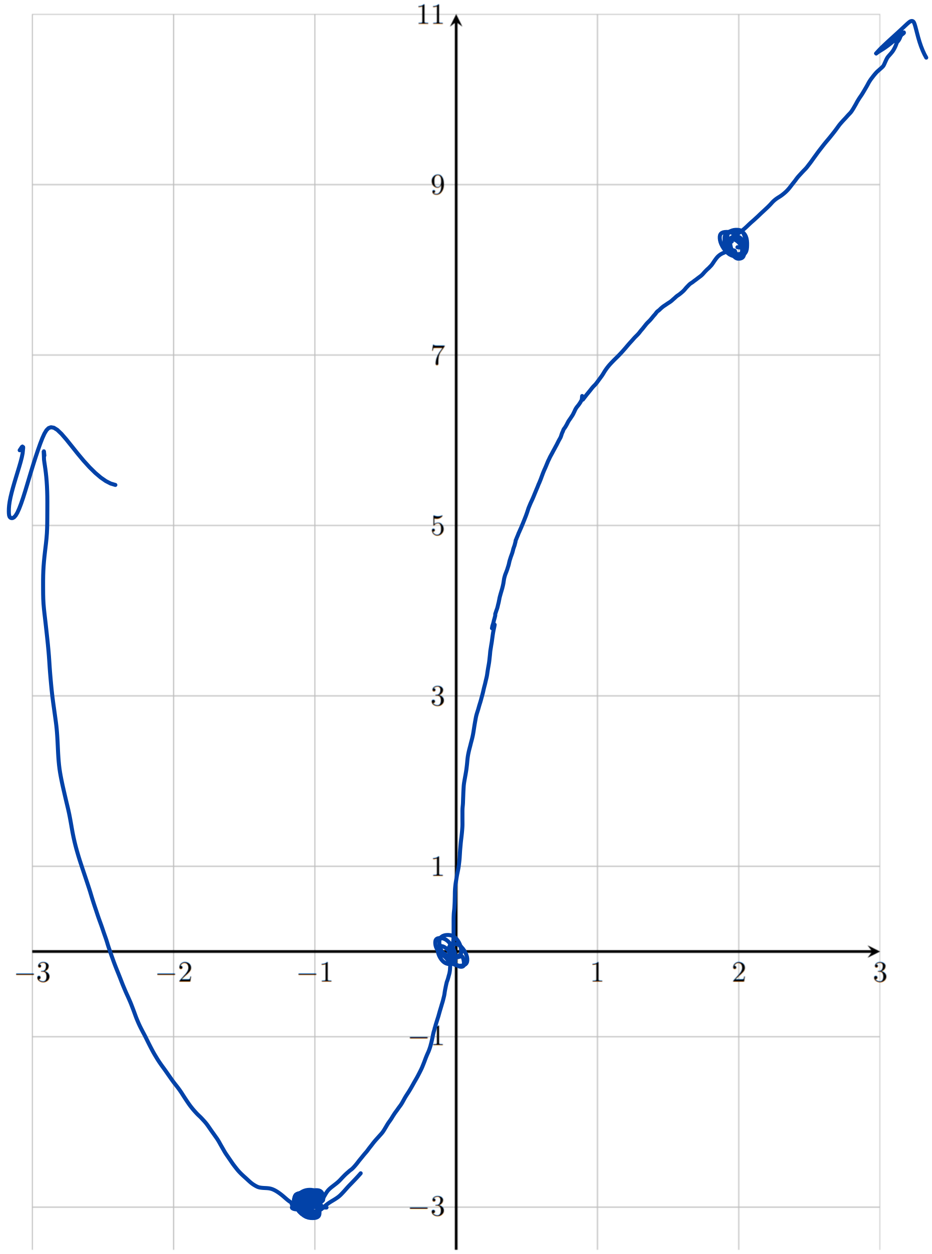
$$f'(1) = \frac{5}{3\sqrt[3]{1}} + \sqrt[3]{1} > 0$$

$$f''(-1) = 6 - \frac{6}{-1} > 0$$

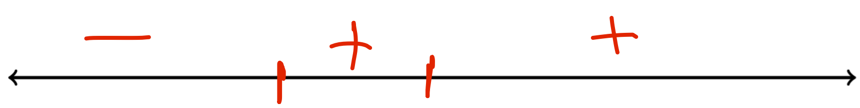
$$f''(1) = 6 - 10 < 0$$

$$f''(8) = 24 - \frac{24}{2} > 0$$

Graph of $f(x) = x^{\frac{1}{3}}(x+4)$



$f'(x)$:



$f''(x)$: